First semestral backpaper exam 2017
B.Math. (Hons.) IInd year
Algebra III — B.Sury

Answer any SIX questions.

Q 1. Prove that the rings $\mathbb{C}[X]/(X^2+1)$ and $\mathbb{C}[t, t^{-1}]$ are isomorphic..

Q 2. Prove that the norm $N(a+b\omega) = a^2 - ab + b^2$ gives a Euclidean algorithm on $\mathbb{Z}[\omega]$ where $\omega = \frac{-1+\sqrt{3}i}{2}$.

Q 3. Let S be any ring with unity. In the ring $M_2(S)$, give examples of left ideals which are not of the form $M_2(I)$ for any left ideal I of S.

Q 4. Find a maximal ideal of $\mathbb{Z}[X]$ containing 4. Show that such an ideal is not principal.

Q 5. Let M be a free module of rank 2 over a PID. Show that any non-zero submodule $N \neq 0$ of M is free, of rank 1 or 2. Can it be of rank 2?

Q 6. Let F be a finite field. Prove that there exist infinitely many irreducible polynomials in $F[X]$.

Q 7. Given a matrix $M \in M_n(K)$, where K is a field, what is meant by its rational canonical form? Further, by assuming the existence of the rational canonical form, compute the characteristic polynomial of M.